

CONSTRUCTION OF MAXIMAL SURFACES IN THE LORENTZ–MINKOWSKI SPACE

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Abstract. The Björling problem for maximal surfaces in Lorentz–Minkowski space \mathbb{L}^3 has been recently studied by the author together with Alías and Chaves. The present paper is a natural extension of that work, and provides several variations of Björling problem. The main scheme is the following. One starts with a spacelike analytic curve in \mathbb{L}^3 , and asks for the construction of a maximal surface which contains that curve, and satisfies additionally some other geometric condition. The solution of these Björling-type problems are then applied with a twofold purpose: to construct examples of maximal surfaces in \mathbb{L}^3 with prescribed properties, and to classify certain families of maximal surfaces.

1. Introduction

In 1844 Björling [2] asked whether, given an analytic strip in \mathbb{R}^3 , it is possible to construct explicitly a minimal surface in \mathbb{R}^3 containing that strip in its interior. The question, known as **Björling problem** for minimal surfaces, was solved in 1890 by Schwarz [9] by means of a complex variable formula which describes minimal surfaces in terms of analytic strips. That formula happened to be quite useful to establish results about minimal surfaces, as well as to construct particular examples of minimal surfaces in \mathbb{R}^3 with interesting geometric properties. Modern approaches to the Björling problem in Euclidean space can be found in [3, 8].

This classic geometric setting was extended in [1] to the case of maximal surfaces in the Lorentz–Minkowski space \mathbb{L}^3 . A surface in \mathbb{L}^3 is a **maximal surface** provided it has zero mean curvature and its induced metric is Riemannian. The solution to Björling problem in \mathbb{L}^3 states the following.